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Procedia Materials Science 12 (2016) 30 – 35

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**Procedia**  
Materials Science

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6th New Methods of Damage and Failure Analysis of Structural Parts [MDFA]

## Including of Ratio of Fatigue Limits from Bending and Torsion for Estimation Fatigue Life under Cyclic Loading

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### Abstract

The paper presents the estimation of the fatigue life under multiaxial cyclic loading of selected construction materials: two aluminum alloys PA4 (6068) and PA6 (2017A), alloy steel S355JOWP (in past called 10HNAP) and cast iron GGG 40. Calculations were based on three criteria of multiaxial fatigue, which is based on the concept of critical plane and the coefficients present in the expressions for the equivalent stresses are calculated on the basis of classical fatigue limits.

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Selection and peer-review under responsibility of the VŠB - Technical University of Ostrava, Faculty of Metallurgy and Materials Engineering

**Keywords:** fatigue life, multiaxial criteria

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### 1. Introduction

The phenomenon of fatigue of materials and structures is a significant issue of the day. Fatigue occurs in various fields of industry, i.e. aerospace, machinery, mining or transport. The main objective of the bulk of research on predicting the fatigue strength is to identify a method of estimating the fatigue strength already on the stage of design and construction of components of machines and devices. The fatigue criterion in multiaxial loading is based on establishing such equivalent value that would enable comparison of multiaxial load with uniaxial loading. Literature of the subject provides a number of multiaxial fatigue criteria (Kurek and Łagoda, 2012). Such criteria are based on various assumptions and parameters of the fatigue process. A separate group of criteria among them are based on the critical plane concept (Karolczuk and Macha 2005). Some of the criteria include the ratio of bending fatigue to torsional fatigue. The paper discusses estimation of fatigue strength depending on the changing orientation of the critical plane of proportional torsional bending for specific construction materials. The paper also compares the

calculation and experimental results for fatigue strength of specific materials, using three different multiaxial fatigue criteria that take into account the ratio of fatigue limits.

### Nomenclature

$A_\sigma, m_\sigma$	coefficients of regression equation for oscillatory bending
$A_\tau, m_\tau$	coefficients of regression equation for torsion
$B, K$	constants used for selection of specific criterion form
$B_1$	constant depending on material type
$N_{cal}$	calculated number of cycles to failure
$N_{exp}$	experimental number of cycles to failure
$N_f$	number of cycles to failure
$\sigma_a$	amplitude of normal stress induced by bending
$\tau_a$	amplitude of shear stress induced by torsion
$\sigma_{af}$	fatigue limits for bending
$\tau_{af}$	fatigue limits for torsion
$\sigma_{eq}$	equivalent stress
$\sigma_\eta$	normal stress component on the critical plane
$\tau_{\eta s}$	tangent stress component on the critical plane

## 2. Fatigue Strength Algorithm

To estimate calculation fatigue life used standard model, which consists of several stages. The first step includes measurement, generation or calculation of component of stress tensor, according to the following equations:

$$\sigma_{xx}(t) = \sigma_a \sin(\omega t), \quad (1)$$

$$\tau_{xy}(t) = \tau_a \sin(\omega t - \varphi), \quad (2)$$

where:

$\sigma_a$  – amplitude of normal stress induced by bending,  $\tau_a$  – amplitude of shear stress induced by torsion,  $\omega$  – angular frequency,  $\varphi$  – angle of phase shift,  $t$  – time.

In the discussed model, the course of normal stress  $\sigma_{xx}(t)$  refers to stress induced by bending, while  $\tau_{xy}(t)$  refers to torsion-induced stress. The next step involves determination of the orientation angle of the critical plane, which can be done using one of three established methods: weight functions, damage accumulation or variance. In this paper, the orientation of the critical plane was determined using damage accumulation method. If the criterion proposed by Carpinteri (Carpinteri and Spagnoli, 2001) is used, the inclination angle of the critical plane is increased by the angle

$$\beta = \frac{3}{2} \left[ 1 - \left( \frac{1}{B_2} \right)^2 \right] 45^\circ, \quad (3)$$

with respect to the angle determined by maximum normal stress, where:

$$B_2 = \frac{\sigma_{af}}{\tau_{af}}. \quad (4)$$

The relationship (4) was proposed for some selected constructional materials, and the group for which this relationship is constant was determined. In such a case, hypotheses allow to calculate the fatigue life. As for other materials, there is no one universal criterion of fatigue life calculation because it is necessary to include variation of the ratio  $\sigma_a/\tau_a(N_f)$  depending on a number of cycles to the fatigue failure (Kurek and Łagoda 2011).

There is a number of multiaxial fatigue criteria. Here, we are discussing the group based on the critical plane concept. Macha (Macha, 1989) has formulated the criterion of maximum normal and shear stress in fracture plane which can be generalised for the scope of random loading of numerous criteria. The general form can be written down as

$$\sigma_{eq}(t) = B\tau_{\eta s}(t) + K\sigma_{\eta}(t). \quad (5)$$

where: B, K – constants used for selection of specific criterion form (Łagoda and Ogonowski, 2005)

In this paper, in order to verify the highest conformity of results, three different criteria of multiaxial fatigue were used:

1. Criterion in the maximum normal stress plane, in the following form

$$\sigma_{eq}(t) = B_1\tau_{\eta s}(t) + \sigma_{\eta}(t), \quad (6)$$

where:  $B_1$  – constant depending on material type,  $\sigma_{\eta}(t)$  is the course of normal stress orientated at angle  $\alpha$  towards  $\sigma_{xx}$ , expressed by the following equation

$$\sigma_{\eta}(t) = \sigma_{xx}(t) \cos^2 \alpha + \tau_{xy}(t) \sin 2\alpha. \quad (7)$$

Whereas  $\tau_{\eta s}(t)$  is the course of shear stress

$$\tau_{\eta s}(t) = -\frac{1}{2}\sigma_{xx}(t) \sin 2\alpha + \tau_{xy}(t) \cos 2\alpha. \quad (8)$$

2. Criterion in the maximum shear stress plane, in the following form

$$\sigma_{eq}(t) = B_2\tau_{\eta s}(t) + (2 - B_2)\sigma_{\eta}(t), \quad (9)$$

3. Criterion proposed by Carpinteri takes the form of equation (5), where:

$$B = \frac{B_2 - \frac{\sin(90^\circ + 2\beta)}{\cos^2 \beta}}{\frac{\sin 2\beta \sin(90^\circ + 2\beta)}{2 \cos^2 \beta} + \cos(90^\circ + 2\beta)}, \quad (10)$$

$$K = \frac{2 + B \sin 2\beta}{2 \cos^2 \beta}. \quad (11)$$

The final step is the calculation of fatigue strength. For fixed amplitude loadings (cyclical), the fatigue strength is calculated using Basquin's fatigue characteristics, in compliance with the relevant ASTM standard (ASTM E 739–91, 1999). The formula for calculation strength under cyclical loading is expressed as

$$N_{cal} = 10^{A - m \lg \sigma_{eg,a}}. \quad (12)$$

### 3. Analysed materials

The analysis used the results of fatigue tests of the following materials: two aluminium alloys: PA4 (6068) (Niesłony et al, 2013) and PA6 (2017A) (Kardas et al, 2008) , 10HNAP steel (Pawliczek , 2001) and GGG40 cast iron (Muller, 1994). The results were also used to calculate the regression equations for oscillatory bending, as per the ASTM recommendations (ASTM E 739–91, 1999), in the following form

$$\log N_f = A_\sigma + m_\sigma \log \sigma_a \quad (13)$$

For bilateral torsion, the regression equation takes the form of

$$\log N_f = A_\tau + m_\tau \log \tau_a \quad (14)$$

where:  $A_\sigma$ ,  $m_\sigma$ ,  $A_\tau$ ,  $m_\tau$  - coefficients of regression equation for oscillatory bending and bilateral torsion, respectively. Table 1 lists the values of coefficients of regression equation for the analysed materials.

Table 1. Coefficients of regression equation for analysed materials.

Material	Bending		Torsion	
	$A_\sigma$	$m_\sigma$	$A_\tau$	$m_\tau$
PA6	21.87	-7.03	19.94	-6.87
GGG40	32.39	-10.95	35.48	-12.41
10HNAP	30.88	-9.5	25.28	-8.2
PA4	23.8	-8.0	21.4	-7.7

### 4. Verification of proposed criteria

The objective of experimental analysis is to verify the efficiency of the proposed method of estimation of fatigue strength under the applied loadings of bending and torsion. Fig. 1 provides a comparison of calculation strength  $N_{cal}$  with experimental strength  $N_{exp}$  for the analysed materials for the case of calculation of fatigue strength by determining the orientation of the critical plane according to normal stress (6).

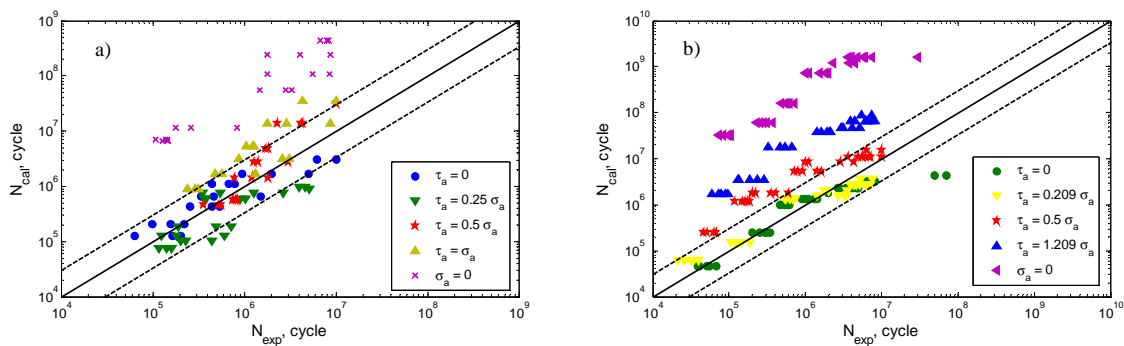


Fig.1. Comparison of obtained calculation strengths with experimental strengths for a) PA6 aluminium alloy b) 10HNAP steel under proportional loading by determining the orientation of the critical plane according to normal stress

Fig. 2 provides a comparison of calculation strength  $N_{cal}$  with experimental strength  $N_{exp}$  for the analysed materials for the case of calculation of fatigue strength by determining the orientation of the critical plane according to shear stress (7).

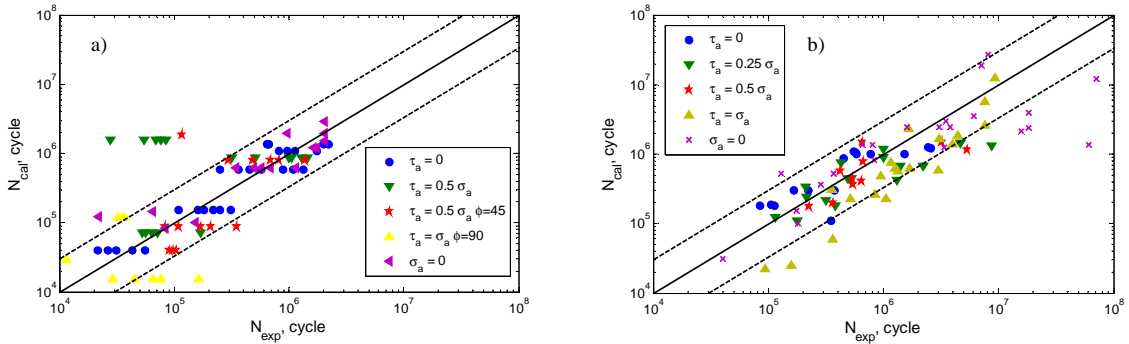


Fig.2. Comparison of obtained calculation strengths with experimental strengths for a) GGG40 cast iron b) PA4 under proportional loading by determining the orientation of the critical plane according to normal stress

Fig. 3 provides a comparison of calculation strength  $N_{cal}$  with experimental strength  $N_{exp}$  for the analysed materials for the case of calculation of fatigue strength using modified criterion of Carpinteri.

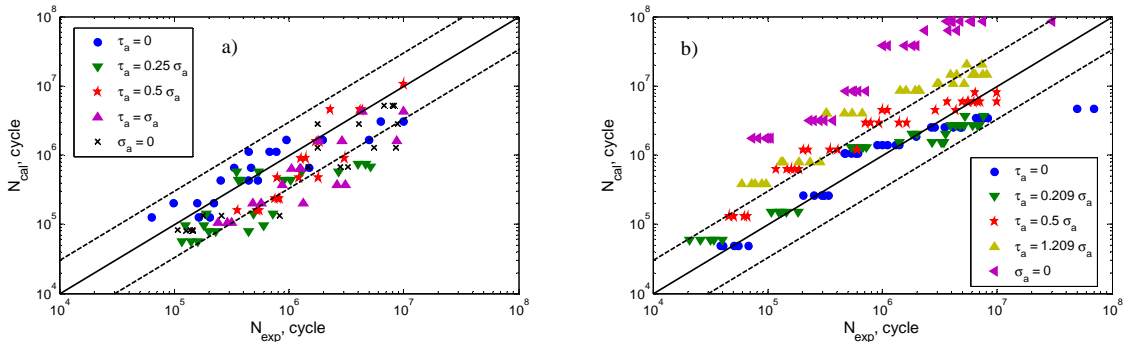


Fig.3. Comparison of obtained calculation strengths with experimental strengths for a) PA6, b) 10HNAP steel under proportional loading by determining the orientation of the critical plane according modified criterion of Carpinteri

## 5. Analysis of obtained results

In order to perform the correct analysis of the fatigue life scatter, the logarithmic dependence of the ratios of experimental and calculation strength should be used. A new method of determination the fatigue life scatter has been proposed by (Walat and Łagoda, 2014) defined as the root mean square error

$$E = \sqrt{\frac{\sum_{i=1}^n \log^2 \frac{N_{exp}}{N_{cal}}}{n}}. \quad (15)$$

Therefore, the scatter can be determined as

$$T = 10^E. \quad (16)$$

Table 2 lists the results of fatigue life scatter for bending and other loadings.

Table 2. Results of fatigue life scatter for analysed materials.

	PA6	PA4	GGG40	10HNAP
$\tau=0$ (pure bending)	1.92	1.94	1.61	2.16
Criterion in the maximum normal stress plane	9.94	9.67	3.57	19.11
Criterion in the maximum shear stress plane	2.29	2.67	3.06	1.95
Modified criterion of Carpinteri	2.59	3.21	3.3	4.63

## 6. Conclusions

Analysing the results of fatigue tests, calculations and data provided in Table 2, it can be concluded that:

1. When the criterion in the maximum normal stress plane is used for all analysed materials, then calculation strength values are inflated in a number of cases (with the exception of cast iron).
2. When the criterion in the maximum shear stress plane and the criterion proposed by Carpinteri are used, the calculation strength for all analysed materials approaches the values of experimental strength. The majority of results fall within the scatter band with the coefficient of 3. The values of fatigue strength scatter for those criteria (Table 2) are lowest, which confirms high conformity of results. The lowest scatter values, however, are obtained for the criterion in the maximum shear stress plane.
3. In the case of cast iron, fatigue strength scatter values are similar for all applied criteria, but not lower than fatigue life scatter values for oscillatory bending.
4. Further verification is required for other materials and other loading conditions.

The project financed from the funds of the National Centre of Science – decision number 2011/01/B/ST8/06850

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